

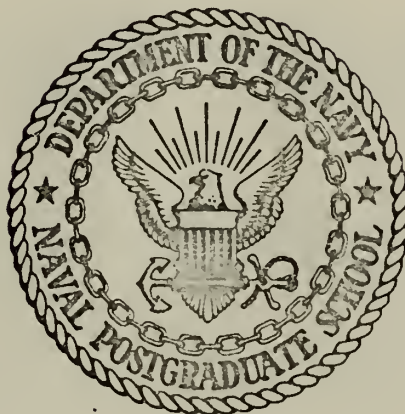
AN INVESTIGATION OF THE COMPUTATION OF  
UPPER CONFIDENCE LEVELS IN A SERIES SYSTEM

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## Monterey, California



# THESIS

AN INVESTIGATION OF THE COMPUTATION OF  
UPPER CONFIDENCE LEVELS IN A SERIES SYSTEM

by

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Thesis Advisor:

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March 1973

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An Investigation of the Computation of  
Upper Confidence Levels in a Series System

by

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## ABSTRACT

A comparison of several techniques is presented for determining upper confidence levels for a system failure rate. A series system of components with exponential failure rates is examined. Classical computational techniques are compared with Bayesian techniques in determining the upper confidence level of a system failure rate. A sensitivity analysis is conducted on several of the parameters as part of the comparison.





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# TABLE OF ABBREVIATIONS

|             |                                                               |
|-------------|---------------------------------------------------------------|
| $\alpha_i$  | Optimistic shape parameter of the $i^{\text{th}}$ component   |
| $\beta_i$   | Optimistic scale parameter of the $i^{\text{th}}$ component   |
| $t_i$       | Total test time of the $i^{\text{th}}$ component              |
| $f_i$       | Number of times the $i^{\text{th}}$ component fails           |
| $\lambda_i$ | Estimate of the failure rate of the $i^{\text{th}}$ component |
| $\lambda_s$ | Estimate of the system failure rate                           |
| $\lambda_u$ | Upper confidence level on the system failure rate             |
| $\gamma$    | Level of significance                                         |
| $k$         | Number of components in the series system                     |
| $R_s$       | System reliability                                            |
| $R_l$       | Lower confidence limit on the system reliability              |



## I. INTRODUCTION

### A. BACKGROUND

Numerous "classic" techniques have been used to compute estimates of failure rate and mean time of failure. From these estimates standard accepted procedures can be applied to establish upper confidence levels (UCL) for the failure rate or lower confidence levels (LCL) for the reliability.

One well established "classic" procedure is to utilize the computational methods set forth in Ref. 3 to determine upper confidence levels on system failure rates. This is the procedure used for all "Classic" and "Semi-Classic" methods presented in this paper.

The application of a "Bayesian" technique may be intuitively appealing to some. Results from previous experiments and testing could be applied apriori to current testing programs to determine failure rate and reliability. By using a prior in such a manner perhaps total system test time, number of component failures, etc., could be reduced, thereby reducing the overall expense involved with system life testing. This would appear to be most appealing when testing systems that are extremely expensive.

The intent of this thesis is to attempt to determine if it would be more advantageous to use some "Bayesian" technique rather than a more traditional "classic" technique to compute upper confidence levels on a system failure rate





when the "prior" for the Bayesian method could be chosen as "optimistically" as one would desire.

## B. SYSTEM ASSUMPTIONS

Since it is the technique of computation of the upper confidence level that is being investigated, the system that is being modeled can be kept simple, yet still realistic.

Even the most complicated system can be broken down to a system of components, connected in series, which the total failure of any one component will result in the mission failure of the system.

Each type component in this series system experiences exponential failure rate. The failure of each type of component is assumed to be independent of the failure of any other type of component. Wearin and wearout are neglected and the failure rate is assumed to be constant. Components are assumed not to ever be "stillborn." To keep the system as simple as possible, each type component exhibits an identical failure rate,  $\lambda_1 = \lambda_i$ ;  $i = 2 \dots k$ ,  $k$  equaling the number of type components that are connected in the series system. The following formulas concerning system failure rate and system reliability are assumed to be valid for this system:

$$\lambda_s = \sum_{i=1}^k \lambda_i$$



$$R_s = R_1 \quad \text{or} \quad R_s = \exp^{-\lambda s}$$

In keeping the system as simple as possible the optimistic "prior" for the Bayesian computations are also chosen to be equal for each type component,  $\alpha_1 = \alpha_i$  and  $\beta_1 = \beta_i$ , for  $i = 1, 2, \dots k$ .

For purposes of comparison, it is assumed that each type of component is placed on test for an equal length of time,  $t_1 = t_i$ , and each type of component experiences the same number of failures during its total test time,  $f_1 = f_i$ . This assumption is modified slightly to be able to examine the system that has only one component failure. A system exhibiting two component failures is also examined.

The data (values of the parameters) have been chosen by the author. They have been intentionally chosen to be computationally simple yet to still exhibit the characteristics of a realistic system.



## II. COMPUTATIONAL METHODS

### A. CLASSIC

Reference 3 is used exclusively for the computations in this method. The computational formulas given in this reference are modified slightly to accommodate the basic assumptions of identical test time, identical number of failures for each component, etc.

$\hat{\lambda}_i$  is a Maximum Likelihood Estimate for the failure rate of the  $i^{\text{th}}$  component. It will be equal to the total number of failures divided by the total test time.

$$\hat{\lambda}_i = \frac{f_i}{t_i} .$$

$\hat{\lambda}_s$ , the system failure rate, will be equal to the sum of the individual component failure rates.  $\hat{\lambda}_s = \sum_{i=1}^k \hat{\lambda}_i$ .

Each component is assumed to fail independently and  $k$  will equal the number of components that are placed together in a series.

$$\lambda_u = \frac{2\hat{\lambda}_s + K'^2 C + (4\hat{\lambda}_s K'^2 C + K'^4 C^2)^{1/2}}{2}$$

$\lambda_u$  will be an upper confidence level on the system failure rate.

$$C = \frac{\sum_{i=1}^k \frac{\hat{\lambda}_i}{t_i}}{\hat{\lambda}_s}$$



The values for  $K'$  are found in Table I. The computational procedure to determine the Beta values for a 90 per cent level of significance are also discussed. Eighty per cent level of significance values are found in Ref. 3.

A formula for the upper confidence level for the system failure rate is also provided when no failures have occurred during the total test time that each component has been allotted. That formula follows:

$$\lambda_u = \frac{K'^2}{n} \sum_{i=1}^k \left( \frac{1}{t_i} \right)$$

where  $n$  equals the number of component terms in the summation.

Substitution of the upper limits on the failure rates obtained will generate corresponding lower confidence limits on reliability; thus

$$R_1 = \exp^{-\lambda_u}.$$

Beta can be calculated from the following formula:

$$X_u - X = \text{Beta}(K) (X_u)^{1/2}$$

$X_u$  is obtained from Chi-Squared tables in Ref. 1 at the desired confidence level with  $2(X+1)$  degrees of freedom where  $X$  is the number of failures and  $K$  is the percentage point of the normal distribution point in the same confidence level.





TABLE I  
BETA VALUES FOR 90 PER CENT CONFIDENCE

| $X$<br>No. of<br>Failures | $X^2$<br>.90, 2(X+1)    | $(1/2X^2 \cdot \frac{X}{\mu} \cdot .90, 2(X+1))$ | Beta<br>Value | $K'$<br>(Beta(1.282)) |
|---------------------------|-------------------------|--------------------------------------------------|---------------|-----------------------|
| 0                         | 4.906                   | 2.3025                                           | 1.18362       | 1.5174                |
| 1                         | 7.779                   | 3.889                                            | 1.14271       | 1.46496               |
| 2                         | 10.645                  | 5.3225                                           | 1.12336       | 1.44015               |
| 3                         | 13.362                  | 6.681                                            | 1.1109        | 1.42417               |
| 4                         | 15.987                  | 7.994                                            | 1.10188       | 1.41261               |
| 5                         | 18.549                  | 9.275                                            | 1.09494       | 1.403713              |
| 10                        | 30.813                  | 15.406                                           | 1.0743        | 1.37725               |
| 20                        | 54.090                  | 27.045                                           | 1.05669       | 1.35468               |
| 30                        | I N T E R P O L A T E D |                                                  |               | 1.34639               |
| 36.5                      | 91.1                    | 45.55                                            | 1.04596       | 1.34092               |

The "Beta Value" listed in this table is germane to Ref. 3 and should not be confused with the  $\beta$  that is the optimistic scale parameter used in the Bayesian simulation.

#### B. BAYESIAN

The Bayesian technique will assume the apriori density to be  $\text{Gamma}(\lambda_i; \alpha_i, \beta_i)$ .  $\alpha_i$  and  $\beta_i$  are the shape and scale parameters. The "prior" can be made "optimistic" if the scale parameter is chosen large in relation to the shape parameter.



The a posteriori density then becomes  $(\lambda_i; \alpha_i + f_i, \beta_i + t_i)$ .  $f_i$  will equal the number of failures of the  $i^{\text{th}}$  type component and  $t_i$  will equal the total test time for the  $i^{\text{th}}$  type component.

The distribution of  $\lambda_s = \sum_{i=1}^k \lambda_i$  will then be determined by computer simulation. Random variates of each  $\lambda_i$  are generated from the Gamma distribution with parameters  $\alpha_i + f_i, \beta_i + t_i$ . A random variate will be generated for each  $\lambda_i, i = 1, \dots, k$ , the number of components in series. The  $\lambda_i$ 's will then be added to determine the series system failure rate.

The process for generating a system failure rate is then repeated 1000 times to yield  $\lambda_{s1}, \lambda_{s2}, \dots, \lambda_{s1000}$ . The 1000 random values of  $\lambda_s$  are then ordered to yield  $\lambda_{s(1)}, \lambda_{s(2)}, \dots, \lambda_{s(1000)}$ .

$\lambda_{su(\gamma)}$  is the "estimate" of the  $(1 - \gamma)^{\text{th}}$  percentile point of the distribution of  $\lambda_s$ .

The Bayesian  $100(1 - \gamma)^{\text{th}}$  upper confidence level for  $\lambda_s$  is then  $\lambda_{s1000(1 - \gamma)}$ .

At the time of this writing a subroutine to generate random variates from the Gamma distribution does not exist in the Naval Postgraduate School computer library. Reference 2 was used to generate random variates from the Gamma distribution with an integer shape parameter.



The generation of random variates with a shape parameter that is not an integer poses a much more complicated problem. Reference 4 has been written to handle this situation for shape parameters between 0.05 and 1.0. Although LT Robinson's approach is relatively untried, it shows a great deal of promise and may possibly be incorporated into the Naval Postgraduate School computer library in the future. It has been used in this writing for comparative purposes for a shape parameter less than 1.0.

### C. SEMI-CLASSIC

Similar to the Classic technique, Ref. 3 is used to compute the upper confidence levels for the system failure rate for this method. The computations will be modified somewhat by adding the identical "optimistic" priors used in the Bayesian technique.

The number of failures per component will be added to  $\alpha_i$  and the total test time per component will be added to  $\beta_i$ . A maximum Likelihood Estimate for the  $i^{\text{th}}$  component's failure is then:

$$\hat{\lambda}_i = \frac{f_i + \alpha_i}{t_i + \beta_i}$$

The system failure rate for this Semi-Classic method is the same as for the Classic method.

$$\hat{\lambda}_s = \sum_{i=1}^k \hat{\lambda}_i$$



The value of C will also change slightly:

$$C = \frac{\hat{\lambda}_i}{\frac{t_i + \beta_i}{\hat{\lambda}_s}}$$

When no failures have occurred, the upper confidence limit on the failure rate now is:

$$\lambda_u = \frac{K^2}{n} \frac{1}{t_i + \beta_i}$$

The computation for the upper confidence level with failure remains the same.

This method is somewhat "Bayesian" in the sense that it is utilizing a prior but it is "Classic" because of the nature of the computational method.





### III. PARAMETERS OF VARIATION

Although the range through which some of the parameters are varied could be more extensive, it does provide the reader with an adequate base for comparing the methods of computation.

|            |                                                                                                                                                                                                                                                                  |
|------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $\alpha_i$ | 1.0, 0.5                                                                                                                                                                                                                                                         |
| $\beta_i$  | 50, 000                                                                                                                                                                                                                                                          |
| $t_i$      | 30, 50, 100 mission units                                                                                                                                                                                                                                        |
| $f_i$      | 1.0, 0.0    The failures are also modified so that $f_1 = 1$ , $f_2 = f_i = 0$ and $f_1 = 1$ , $f_2 = 1$ , $f_3 = f_i$ , $f_i = 0$ . This will provide the reader with the opportunity to compare a system where only one component or only two components fail. |
| $k$        | 1, 2, 5, 10, 30 components                                                                                                                                                                                                                                       |
| $\gamma$   | 0.10                                                                                                                                                                                                                                                             |

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#### IV. COMPARISONS OF SYSTEM FAILURE RATE UPPER CONFIDENCE LEVELS

The following 12 tables provide the reader with upper confidence levels computed from the same data by the three different methods: Classic, Semi-Classic, and Bayesian.

A program to generate random variates from a Gamma distribution with a shape parameter that was greater than 1.0 but not an integer was not available; therefore UCL's for the Bayesian simulation could not be computed for the case when the shape parameter was less than 1.0 and a failure existed for the  $i^{\text{th}}$  component.

All calculations were conducted on the IBM 360 computer with the exception of the Classic method with zero failures, whose calculations were computed on a desk calculator.

For each three-line block of numbers the reader may compare the three methods' upper confidence levels computed from the same arguments. To see how a change in the scale parameter, the number of times a component fails, or the number of components in series is reflected in the upper confidence level, the reader merely moves to the right or down in the table. If the reader wants to see how a change in the shape parameter or a change in the amount of time a component is on test is reflected, he must go to another table.



TABLE II

SYSTEM FAILURE RATE UPPER CONFIDENCE LEVELS  $\text{ALFA}_i = 1.0$ 

| Time <sub>i</sub> | Beta <sub>i</sub> | Fail <sub>i</sub> | k               | 1       | 2       | 5       | 10      | 30       |
|-------------------|-------------------|-------------------|-----------------|---------|---------|---------|---------|----------|
| 30                | 50                | 0                 | CLASSIC         | .076750 | .076750 | .076750 | .076750 | .076750  |
|                   |                   |                   | SEMI<br>CLASSIC | .048790 | .066531 | .115937 | .192571 | .478729  |
|                   |                   |                   | BAYESIAN        | .026661 | .046493 | .097500 | .172700 | .460052  |
|                   |                   | 1                 | CLASSIC         | .130107 | .174417 | .309166 | .513524 | 1.276610 |
|                   |                   |                   | SEMI<br>CLASSIC | .067770 | .101229 | .194152 | .339753 | .891556  |
|                   |                   |                   | BAYESIAN        | .046493 | .083120 | .172700 | .321967 | .868478  |
|                   |                   | 0                 | CLASSIC         | .076750 | .076750 | .076750 | .076750 | .076750  |
|                   |                   |                   | SEMI<br>CLASSIC | .030025 | .040942 | .071346 | .118505 | .294603  |
|                   |                   |                   | BAYESIAN        | .016407 | .028611 | .060000 | .106277 | .283109  |
|                   | 100               | 1                 | CLASSIC         | .130107 | .174417 | .309166 | .513524 | 1.276610 |
|                   |                   |                   | SEMI<br>CLASSIC | .041705 | .062295 | .119478 | .209079 | .548650  |
|                   |                   |                   | BAYESIAN        | .028611 | .051577 | .106277 | .198133 | .534447  |





TABLE III

SYSTEM FAILURE RATE UPPER CONFIDENCE LEVELS  $\text{ALFA}_i = 1.0$ 

| Time <sub>i</sub> | Beta <sub>i</sub> | Fail <sub>i</sub> | k            | 1       | 2       | 5       | 10      | 30      |
|-------------------|-------------------|-------------------|--------------|---------|---------|---------|---------|---------|
| 50                | 50                | 0                 | CLASSIC      | .046050 | .046050 | .046050 | .046050 | .046050 |
|                   |                   |                   | SEMI CLASSIC | .039032 | .053225 | .092750 | .154057 | .382984 |
|                   |                   |                   | BAYESIAN     | .021329 | .037194 | .078000 | .138160 | .368041 |
|                   |                   | 1                 | CLASSIC      | .078064 | .106450 | .185500 | .308114 | .765967 |
|                   |                   |                   | SEMI CLASSIC | .054216 | .080983 | .155321 | .271802 | .713245 |
|                   |                   |                   | BAYESIAN     | .037194 | .067050 | .138160 | .257573 | .694782 |
|                   | 100               | 0                 | CLASSIC      | .046050 | .046050 | .046050 | .046050 | .046050 |
|                   |                   |                   | SEMI CLASSIC | .026021 | .035483 | .061833 | .102705 | .255322 |
|                   |                   |                   | BAYESIAN     | .014219 | .024796 | .052000 | .092107 | .245361 |
|                   |                   | 1                 | CLASSIC      | .078064 | .106450 | .185500 | .308114 | .765967 |
|                   |                   |                   | SEMI CLASSIC | .036144 | .053989 | .103548 | .181202 | .475497 |
|                   |                   |                   | BAYESIAN     | .024796 | .044700 | .092107 | .171715 | .463188 |





TABLE IV

SYSTEM FAILURE RATE UPPER CONFIDENCE LEVELS  $\text{ALFA}_i = 1.0$ 

| Time <sub>i</sub> | Beta <sub>i</sub> | Fail <sub>i</sub> | k               | 1       | 2       | 5       | 10      | 30      |
|-------------------|-------------------|-------------------|-----------------|---------|---------|---------|---------|---------|
| 100               | 50                | 0                 | CLASSIC         | .023025 | .023025 | .023025 | .023025 | .023025 |
|                   |                   |                   | SEMI<br>CLASSIC | .026021 | .035483 | .061833 | .102705 | .255322 |
|                   |                   |                   | BAYESIAN        | .014219 | .024796 | .052000 | .092107 | .245361 |
|                   |                   | 1                 | CLASSIC         | .039032 | .053225 | .092750 | .154057 | .382984 |
|                   |                   |                   | SEMI<br>CLASSIC | .036144 | .053989 | .103548 | .181202 | .475497 |
|                   |                   |                   | BAYESIAN        | .024796 | .044700 | .092107 | .171715 | .463188 |
|                   | 100               | 0                 | CLASSIC         | .023025 | .023025 | .023025 | .023025 | .023025 |
|                   |                   |                   | SEMI<br>CLASSIC | .019516 | .026612 | .046375 | .077029 | .191492 |
|                   |                   |                   | BAYESIAN        | .010665 | .018597 | .039000 | .069080 | .184021 |
|                   |                   | 1                 | CLASSIC         | .039032 | .053225 | .092750 | .154057 | .382984 |
|                   |                   |                   | SEMI<br>CLASSIC | .027108 | .040492 | .077661 | .135901 | .356623 |
|                   |                   |                   | BAYESIAN        | .018597 | .033525 | .069080 | .128787 | .347391 |



TABLE V

SYSTEM FAILURE RATE UPPER CONFIDENCE LEVELS  $\text{ALFA}_i = 0.5$ 

| Time <sub>i</sub> | Beta <sub>i</sub> | Fail <sub>i</sub> | k            | 1       | 2       | 5       | 10      | 30       |
|-------------------|-------------------|-------------------|--------------|---------|---------|---------|---------|----------|
| 30                | 50                | 0                 | CLASSIC      | .076750 | .076750 | .076750 | .076750 | .076750  |
|                   |                   |                   | SEMI CLASSIC | .038477 | .047646 | .073919 | .114634 | .264620  |
|                   |                   |                   | BAYESIAN     | .018439 | .028627 | .057292 | .101570 | .258735  |
|                   |                   | 1                 | CLASSIC      | .130107 | .177417 | .309166 | .513524 | 1.276610 |
|                   |                   |                   | SEMI CLASSIC | .058481 | .084230 | .155671 | .267077 | .686737  |
|                   |                   |                   | BAYESIAN     | *****   | *****   | *****   | *****   | *****    |
|                   | 100               | 0                 | CLASSIC      | .076750 | .076750 | .076750 | .076750 | .076750  |
|                   |                   |                   | SEMI CLASSIC | .023678 | .029321 | .045488 | .070544 | .162843  |
|                   |                   |                   | BAYESIAN     | .011347 | .017617 | .035257 | .062505 | .159222  |
|                   |                   | 1                 | CLASSIC      | .130107 | .177417 | .309166 | .513524 | 1.276610 |
|                   |                   |                   | SEMI CLASSIC | .035988 | .051834 | .095797 | .164355 | .422608  |
|                   |                   |                   | BAYESIAN     | *****   | *****   | *****   | *****   | *****    |



TABLE VI

SYSTEM FAILURE RATE UPPER CONFIDENCE LEVELS  $\text{ALFA}_i = 0.5$

| Time <sub>i</sub> | Beta <sub>i</sub> | Fail <sub>i</sub> | k               | 1       | 2       | 5       | 10      | 30      |
|-------------------|-------------------|-------------------|-----------------|---------|---------|---------|---------|---------|
| 50                | 50                | 0                 | CLASSIC         | .046050 | .046050 | .046050 | .046050 | .046050 |
|                   |                   |                   | SEMI<br>CLASSIC | .030078 | .038117 | .059135 | .091708 | .211696 |
|                   |                   |                   | BAYESIAN        | .014751 | .022902 | .045834 | .081256 | .206988 |
|                   |                   | 1                 | CLASSIC         | .078064 | .106450 | .185500 | .308114 | .765967 |
|                   |                   |                   | SEMI<br>CLASSIC | .046785 | .067384 | .124537 | .213661 | .549390 |
|                   |                   |                   | BAYESIAN        | *****   | *****   | *****   | *****   | *****   |
|                   | 100               | 0                 | CLASSIC         | .046050 | .046050 | .046050 | .046050 | .046050 |
|                   |                   |                   | SEMI<br>CLASSIC | .020521 | .025411 | .039423 | .061138 | .141131 |
|                   |                   |                   | BAYESIAN        | .009834 | .015268 | .030556 | .054171 | .137992 |
|                   |                   | 1                 | CLASSIC         | .078064 | .106450 | .185500 | .308114 | .765967 |
|                   |                   |                   | SEMI<br>CLASSIC | .031190 | .044923 | .083024 | .142441 | .366260 |
|                   |                   |                   | BAYESIAN        | **      | *****   | *****   | *****   | *****   |





TABLE VII

SYSTEM FAILURE RATE UPPER CONFIDENCE LEVELS  $\text{ALFA}_i = 0.5$

| Time <sub>i</sub> | Beta <sub>i</sub> | Fail <sub>i</sub> | k            | 1       | 2       | 5       | 10      | 30      |
|-------------------|-------------------|-------------------|--------------|---------|---------|---------|---------|---------|
| 100               | 50                | 0                 | CLASSIC      | .023025 | .023025 | .023025 | .023025 | .023025 |
|                   |                   |                   | SEMI CLASSIC | .020521 | .025411 | .039423 | .061138 | .141131 |
|                   |                   |                   | BAYESIAN     | .009834 | .015268 | .030556 | .054171 | .137992 |
|                   |                   | 1                 | CLASSIC      | .039032 | .053225 | .092750 | .154057 | .382984 |
|                   |                   |                   | SEMI CLASSIC | .031190 | .044923 | .083024 | .142441 | .366260 |
|                   |                   |                   | BAYESIAN     | *****   | *****   | *****   | *****   | *****   |
|                   | 100               | 0                 | CLASSIC      | .023025 | .023025 | .023025 | .023025 | .023025 |
|                   |                   |                   | SEMI CLASSIC | .015391 | .019058 | .029568 | .045854 | .105848 |
|                   |                   |                   | BAYESIAN     | .007376 | .011451 | .022917 | .040628 | .103494 |
|                   |                   | 1                 | CLASSIC      | .039032 | .053225 | .092750 | .154057 | .382984 |
|                   |                   |                   | SEMI CLASSIC | .023390 | .033692 | .062268 | .106831 | .274695 |
|                   |                   |                   | BAYESIAN     | *****   | *****   | *****   | *****   | *****   |





TABLE VIII

MODIFIED UPPER CONFIDENCE LEVELS ON SYSTEM FAILURE RATES  $ALFA_i = 1.0$

| Time <sub>i</sub> | Beta <sub>i</sub> | Fail <sub>i</sub>               | k               | 1       | 2       | 5       | 10      | 30      |
|-------------------|-------------------|---------------------------------|-----------------|---------|---------|---------|---------|---------|
| 30                | 50                | f <sub>1</sub> = 1              | CLASSIC         | .130107 | .130107 | .130107 | .130107 | .130107 |
|                   |                   | f <sub>2</sub> = f <sub>i</sub> | SEMI<br>CLASSIC | .067770 | .136695 | .192806 | .279285 | .594334 |
|                   |                   | f <sub>i</sub> = 0              | BAYESIAN        | .046493 | .065268 | .114501 | .187769 | .472251 |
|                   |                   | f <sub>1</sub> = 1              | CLASSIC         | *****   | .177417 | .177417 | .177417 | .177417 |
|                   |                   | f <sub>2</sub> = 1              | SEMI<br>CLASSIC | *****   | .152770 | .207180 | .292108 | .604415 |
|                   |                   | f <sub>3</sub> = f <sub>i</sub> |                 |         |         |         |         |         |
|                   |                   | f <sub>i</sub> = 0              | BAYESIAN        | *****   | .083812 | .129434 | .202936 | .489156 |
|                   |                   |                                 |                 |         |         |         |         |         |
|                   |                   |                                 |                 |         |         |         |         |         |
|                   | 100               | f <sub>1</sub> = 1              | CLASSIC         | .130107 | .130107 | .130107 | .130107 | .130107 |
|                   |                   | f <sub>2</sub> = f <sub>i</sub> | SEMI<br>CLASSIC | .041705 | .113440 | .150096 | .206547 | .410319 |
|                   |                   | f <sub>i</sub> = 0              | BAYESIAN        | .028611 | .040165 | .070462 | .115550 | .290616 |
|                   |                   | f <sub>1</sub> = 1              | CLASSIC         | *****   | .177417 | .177417 | .177417 | .177417 |
|                   |                   | f <sub>2</sub> = 1              | SEMI<br>CLASSIC | *****   | .122974 | .158538 | .213918 | .415675 |
|                   |                   | f <sub>3</sub> = f <sub>i</sub> |                 |         |         |         |         |         |
|                   |                   | f <sub>i</sub> = 0              | BAYESIAN        | *****   | .051577 | .079652 | .124883 | .301019 |



TABLE IX

MODIFIED UPPER CONFIDENCE LEVELS ON SYSTEM FAILURE RATES  $\text{ALFA}_i = 1.0$ 

| Time <sub>i</sub> | Beta <sub>i</sub> | Fail <sub>i</sub> | k               | 1       | 2       | 5       | 10      | 30      |
|-------------------|-------------------|-------------------|-----------------|---------|---------|---------|---------|---------|
| 50                | 50                | $f_1 = 1$         | CLASSIC         | .078064 | .078064 | .078064 | .078064 | .078064 |
|                   |                   | $f_2 = f_i$       | SEMI<br>CLASSIC | .054216 | .093570 | .136890 | .203823 | .449300 |
|                   |                   | $f_i = 0$         | BAYESIAN        | .037194 | .052215 | .091601 | .150216 | .377801 |
|                   |                   | $f_1 = 1$         | CLASSIC         | *****   | .106450 | .106450 | .106450 | .106450 |
|                   |                   | $f_2 = 1$         | SEMI<br>CLASSIC | *****   | .106450 | .148479 | .214278 | .457804 |
|                   |                   | $f_3 = f_i$       | BAYESIAN        | *****   | .067050 | .103548 | .162349 | .391324 |
|                   |                   | $f_i = 0$         |                 |         |         |         |         |         |
|                   |                   | $f_1 = 1$         | CLASSIC         | .078064 | .078064 | .078064 | .078064 | .078064 |
|                   |                   | $f_2 = f_i$       | SEMI<br>CLASSIC | .036144 | .078064 | .108433 | .155206 | .325173 |
|                   |                   | $f_i = 0$         | BAYESIAN        | .024796 | .034810 | .061067 | .100144 | .251867 |
|                   | 100               | $f_1 = 1$         | CLASSIC         | *****   | .106450 | .106450 | .106450 | .106450 |
|                   |                   | $f_2 = 1$         | SEMI<br>CLASSIC | *****   | .086603 | .116048 | .161966 | .330403 |
|                   |                   | $f_3 = f_i$       | BAYESIAN        | *****   | .044700 | .069032 | .108232 | .260883 |
|                   |                   | $f_i = 0$         |                 |         |         |         |         |         |





TABLE X

MODIFIED UPPER CONFIDENCE LEVELS ON SYSTEM FAILURE RATES  $\text{ALFA}_i = 1.0$ 

| Time <sub>i</sub> | Beta <sub>i</sub> | Fail <sub>i</sub> | k               | 1       | 2       | 5       | 10      | 30      |
|-------------------|-------------------|-------------------|-----------------|---------|---------|---------|---------|---------|
| 100               | 50                | $f_1 = 1$         | CLASSIC         | .039032 | .039032 | .039032 | .039032 | .039032 |
|                   |                   | $f_2 = f_i$       | SEMI<br>CLASSIC | .036144 | .054216 | .082107 | .125363 | .285135 |
|                   |                   | $f_i = 0$         | BAYESIAN        | .024796 | .034810 | .061067 | .100144 | .251867 |
|                   |                   | $f_1 = 1$         | CLASSIC         | *****   | .053225 | .053225 | .053225 | .053225 |
|                   |                   | $f_2 = 1$         | SEMI<br>CLASSIC | *****   | .062739 | .089830 | .132403 | .291025 |
|                   |                   | $f_3 = f_i$       | BAYESIAN        | *****   | .044700 | .069032 | .108232 | .260883 |
|                   |                   | $f_i = 0$         |                 |         |         |         |         |         |
|                   |                   | $f_1 = 1$         | CLASSIC         | .039032 | .039032 | .039032 | .039032 | .039032 |
|                   |                   | $f_2 = f_i$       | SEMI<br>CLASSIC | .027108 | .046785 | .068445 | .101911 | .224650 |
|                   |                   | $f_i = 0$         | BAYESIAN        | .018597 | .026107 | .045800 | .075108 | .188900 |
|                   |                   | $f_1 = 1$         | CLASSIC         | *****   | .053225 | .053225 | .053225 | .053225 |
|                   |                   | $f_2 = 1$         | SEMI<br>CLASSIC | *****   | .053225 | .074240 | .107139 | .228902 |
|                   |                   | $f_3 = f_i$       | BAYESIAN        | *****   | .033535 | .051774 | .081174 | .195662 |
|                   |                   | $f_i = 0$         |                 |         |         |         |         |         |
| 100               | 100               | $f_1 = 1$         | CLASSIC         | .039032 | .039032 | .039032 | .039032 | .039032 |
|                   |                   | $f_2 = f_i$       | SEMI<br>CLASSIC | .027108 | .046785 | .068445 | .101911 | .224650 |
|                   |                   | $f_i = 0$         | BAYESIAN        | .018597 | .026107 | .045800 | .075108 | .188900 |
|                   |                   | $f_1 = 1$         | CLASSIC         | *****   | .053225 | .053225 | .053225 | .053225 |
|                   |                   | $f_2 = 1$         | SEMI<br>CLASSIC | *****   | .053225 | .074240 | .107139 | .228902 |
|                   |                   | $f_3 = f_i$       | BAYESIAN        | *****   | .033535 | .051774 | .081174 | .195662 |
|                   |                   | $f_i = 0$         |                 |         |         |         |         |         |
|                   |                   | $f_1 = 1$         | CLASSIC         | .039032 | .039032 | .039032 | .039032 | .039032 |
|                   |                   | $f_2 = f_i$       | SEMI<br>CLASSIC | .027108 | .046785 | .068445 | .101911 | .224650 |
|                   |                   | $f_i = 0$         | BAYESIAN        | .018597 | .026107 | .045800 | .075108 | .188900 |
|                   |                   | $f_1 = 1$         | CLASSIC         | *****   | .053225 | .053225 | .053225 | .053225 |
|                   |                   | $f_2 = 1$         | SEMI<br>CLASSIC | *****   | .053225 | .074240 | .107139 | .228902 |
|                   |                   | $f_3 = f_i$       | BAYESIAN        | *****   | .033535 | .051774 | .081174 | .195662 |
|                   |                   | $f_i = 0$         |                 |         |         |         |         |         |



TABLE XI

MODIFIED UPPER CONFIDENCE LEVELS ON SYSTEM FAILURE RATES  $\text{ALFA}_i = 0.5$ 

| Time <sub>i</sub> | Beta <sub>i</sub> | Fail <sub>i</sub> | k               | 1       | 2       | 5       | 10      | 30      |
|-------------------|-------------------|-------------------|-----------------|---------|---------|---------|---------|---------|
| 30                | 50                | $f_1 = 1$         | CLASSIC         | .130107 | .130107 | .130107 | .130107 | .130107 |
|                   |                   | $f_2 = f_i$       | SEMI<br>CLASSIC | .054810 | .116621 | .146407 | .192806 | .361256 |
|                   |                   | $f_i = 0$         | BAYESIAN        | *****   | *****   | *****   | *****   | *****   |
|                   |                   | $f_1 = 1$         | CLASSIC         | *****   | .177417 | .177417 | .177417 | .177417 |
|                   |                   | $f_2 = 1$         | SEMI<br>CLASSIC | *****   | .133609 | .162117 | .207180 | .373106 |
|                   |                   | $f_3 = f_i$       | BAYESIAN        | *****   | *****   | *****   | *****   | *****   |
|                   |                   | $f_i = 0$         |                 |         |         |         |         |         |
|                   |                   |                   |                 |         |         |         |         |         |
|                   |                   |                   |                 |         |         |         |         |         |
|                   | 100               | $f_1 = 1$         | CLASSIC         | .130107 | .130107 | .130107 | .130107 | .130107 |
|                   |                   | $f_2 = f_i$       | SEMI<br>CLASSIC | .035988 | .100392 | .119775 | .150096 | .259836 |
|                   |                   | $f_i = 0$         | BAYESIAN        | *****   | *****   | *****   | *****   | *****   |
|                   |                   | $f_1 = 1$         | CLASSIC         | *****   | .177417 | .177417 | .177417 | .177417 |
|                   |                   | $f_2 = 1$         | SEMI<br>CLASSIC | *****   | .110467 | .129082 | .158538 | .266507 |
|                   |                   | $f_3 = f_i$       | BAYESIAN        | *****   | *****   | *****   | *****   | *****   |
|                   |                   | $f_i = 0$         |                 |         |         |         |         |         |
|                   |                   |                   |                 |         |         |         |         |         |
|                   |                   |                   |                 |         |         |         |         |         |





TABLE XII

MODIFIED UPPER CONFIDENCE LEVELS ON SYSTEM FAILURE RATES  $\text{ALFA}_i = 0.5$ 

| Time <sub>i</sub> | Beta <sub>i</sub> | Fail <sub>i</sub> | k               | 1       | 2       | 5       | 10      | 30      |
|-------------------|-------------------|-------------------|-----------------|---------|---------|---------|---------|---------|
| 50                | 50                | $f_1 = 1$         | CLASSIC         | .078064 | .078064 | .078064 | .078064 | .078064 |
|                   |                   | $f_2 = f_i$       | SEMI<br>CLASSIC | .046785 | .078064 | .101068 | .136890 | .267480 |
|                   |                   | $f_i = 0$         | BAYESIAN        | *****   | *****   | *****   | *****   | *****   |
|                   |                   | $f_1 = 1$         | CLASSIC         | *****   | .106450 | .106450 | .106450 | .106450 |
|                   |                   | $f_2 = 1$         | SEMI<br>CLASSIC | *****   | .091662 | .113665 | .148479 | .277238 |
|                   |                   | $f_i = 0$         | BAYESIAN        | *****   | *****   | *****   | *****   | *****   |
|                   | 100               | $f_1 = 1$         | CLASSIC         | .078064 | .078064 | .078064 | .078064 | .078064 |
|                   |                   | $f_2 = f_i$       | SEMI<br>CLASSIC | .031190 | .067210 | .083321 | .108433 | .199486 |
|                   |                   | $f_i = 0$         | BAYESIAN        | *****   | *****   | *****   | *****   | *****   |
|                   |                   | $f_1 = 1$         | CLASSIC         | *****   | .106450 | .106450 | .106450 | .106450 |
|                   |                   | $f_2 = 1$         | SEMI<br>CLASSIC | *****   | .076234 | .091662 | .116048 | .205707 |
|                   |                   | $f_i = 0$         | BAYESIAN        | *****   | *****   | *****   | *****   | *****   |



TABLE XIII

MODIFIED UPPER CONFIDENCE LEVELS ON SYSTEM FAILURE RATES ALFA = 0.5

| Time <sub>i</sub> | Beta <sub>i</sub> | Fail <sub>i</sub> | k               | 1       | 2       | 5       | 10      | 30      |
|-------------------|-------------------|-------------------|-----------------|---------|---------|---------|---------|---------|
| 100               | 50                | $f_1 = 1$         | CLASSIC         | .039032 | .039032 | .039032 | .039032 | .039032 |
|                   |                   | $f_2 = f_i$       | SEMI<br>CLASSIC | .031190 | .044243 | .059039 | .082107 | .166657 |
|                   |                   | $f_i = 0$         | BAYESIAN        | *****   | *****   | *****   | *****   | *****   |
|                   |                   | $f_1 = 1$         | CLASSIC         | *****   | .053225 | .053225 | .053225 | .053225 |
|                   |                   | $f_2 = 1$         | SEMI<br>CLASSIC | *****   | .053225 | .067384 | .089830 | .173283 |
|                   |                   | $f_3 = f_i$       | BAYESIAN        | *****   | *****   | *****   | *****   | *****   |
|                   |                   | $f_i = 0$         |                 |         |         |         |         |         |
|                   |                   |                   |                 |         |         |         |         |         |
|                   |                   |                   |                 |         |         |         |         |         |
|                   | 100               | $f_1 = 1$         | CLASSIC         | .039032 | .039032 | .039032 | .039032 | .039032 |
|                   |                   | $f_2 = f_i$       | SEMI<br>CLASSIC | .023392 | .039032 | .050534 | .068445 | .133740 |
|                   |                   | $f_i = 0$         | BAYESIAN        | *****   | *****   | *****   | *****   | *****   |
|                   |                   | $f_1 = 1$         | CLASSIC         | *****   | .053225 | .053225 | .053225 | .053225 |
|                   |                   | $f_2 = 1$         | SEMI<br>CLASSIC | *****   | .045831 | .056833 | .074240 | .138619 |
|                   |                   | $f_3 = f_i$       | BAYESIAN        | *****   | *****   | *****   | *****   | *****   |
|                   |                   | $f_i = 0$         |                 |         |         |         |         |         |
|                   |                   |                   |                 |         |         |         |         |         |
|                   |                   |                   |                 |         |         |         |         |         |



## V. CONCLUSIONS

A "crossover" point in this discussion will be defined as the point or general area at which an upper confidence level that was previously lower than one with which it was being compared becomes higher.

The Bayesian method will be compared with the Classic and the Semi-Classic will be compared with Classic. The Bayesian and the Semi-Classic appear to behave similarly and remarks concerning this similarity will be mentioned.

### A. UNMODIFIED COMPARISONS, $\text{ALFA} = 1.0$

The first three tables deal with a shape parameter of 1.0 and a system of components that either all of the components experience a failure or none of the components experience a failure.

In the cases where none of the components fail, a crossover point is exhibited in every case when five or more components are in series, regardless of how optimistic the scale parameter becomes or how long the test time is extended. When every component fails, the Bayesian and Semi-Classic systems do not crossover. In both cases the values of the upper confidence level for the Bayesian and the Semi-Classic methods become closer together as the number of components increases.





## B. UNMODIFIED COMPUTATIONS, ALFA = 0.5

The upper confidence levels for the second three tables (less the Classic) are computed with a more optimistic prior shape parameter ( $\alpha = 0.5$ ) so naturally the UCL's for these two methods are lower than those shown in the first three tables.

Crossover points are much the same as in the case when  $\alpha = 1.0$ . The Bayesian and Semi-Classic methods crossover when none of the components fail; only the crossover point occurs when about 10 components are placed in series. The Semi-Classic does not crossover when all components fail (the Bayesian is not examined due to the non-existence of a Gamma random variate generator with a non-integer shape parameter).

## C. MODIFIED COMPUTATIONS, ALFA = 1.0

The third three tables provide a much more interesting comparison. Here the situation where only one (or two) component(s) in the system fail. The Semi-Classic and the Bayesian techniques will crossover the Classic system in every case regardless of how optimistic the "priors" are or how long the components are tested. When the priors are more optimistic and/or the test time is larger, then it takes a larger number of components in series for the crossover to occur. This point may occur with as few as two components or as many as between 10 and





30 components. The Bayesian UCL's are strictly lower than the Semi-Classic UCL's.

#### D. MODIFIED COMPUTATIONS, $\text{ALFA} = 0.5$

In the last three tables only the Semi-Classic and the Classic methods can be compared for the Bayesian cannot be simulated for this case (non-integer).

The Semi-Classic technique is using a more optimistic prior so its UCL's are lower than before but again in every case it will crossover with the Classic method.

Generally one could conclude that as long as the number of components that are in series in a system are few, the Bayesian approach may yield a lower value of an upper confidence level for the system failure rate, provided the priors can be chosen optimistically. If the system is complex enough that more than "a few" components must be placed in series then the Classic system appears to yield the lowest values of upper confidence levels. The Semi-Classic technique would only be useful if the priors were optimistic, the number of components in series were few, and the optimistic priors were non-integers.



# COMPUTER PROGRAM FOR THE CLASSIC METHOD

```

C THE VARIABLE DEFINITIONS FOR THIS PROGRAM FOLLOW:
C TIME THE NUMBER OF MISSION UNITS EACH COMPONENT IS UNDER TEST
C FAIL THE NUMBER OF TIMES THAT EACH COMPONENT WILL FAIL DURING THE TOTAL TEST TIME
C XK THE NUMBER OF COMPONENTS THAT ARE IN THE SERIES SYSTEM
C XLAMI INDIVIDUAL COMPONENT FAILURE RATE
C XLAMS SYSTEM FAILURE RATE
C XKPRI A VALUE TAKEN FROM TABLE I (BETA VALUES)
C

XK=1.0
XKPRI=1.469496
7 TIME=30.0
10 C=0.0
FAIL=1.0
XLAMUP=0.0
XLAMI=FAIL/TIME
XLAMS=XK*XLAMI
C=(XK*(XLAMI/TIME))/XLAMS
XLAMUP=(2.0*XLAMS+(XKPRI**2)*C+SQRT(((4.0*XLAMS)*(XKPRI**2)*C)+(XKPRI**4)*(C**2)))/2.0
11 WRITE(6,101)FAIL,XK,TIME,XLAMUP
101 FORMAT(' ',5X,'FAIL=',F7.3,5X,'XK=',F7.3,5X,'TIME=',F7.3,5X,'XLAMUP = ',F10.6,/)
2.3 TIME=TIME+20.0
102 IF(TIME.LE.51.0) GO TO 10
TIME=TIME+30.0
103 IF(TIME.LE.101.0) GO TO 10
XK=XK+1.0
XKPRI=1.44015
104 IF(XK.LE.2.3) GO TO 7
XK=XK+2.0
XKPRI=1.403713
105 IF(XK.LT.5.5) GO TO 7
XK=XK+2.0
XKPRI=1.37725
106 IF(XK.LT.10.5) GO TO 7
XK=XK+15.0
XKPRI=1.34092
107 IF(XK.LT.30.5) GO TO 7
STOP
END

```



# COMPUTER PROGRAM FOR THE SEMI-CLASSIC METHOD

```

C      THIS SYSTEM IS LABELED "SEMI-CLASSIC" BECAUSE IT
C      UTILIZES THE "PRIORS" THAT WERE INPUTS TO THE BAYES-
C      IAN TECHNIQUE BUT THE COMPUTATIONS ARE PERFORMED THE
C      THE SAME WAY AS IN THE "CLASSIC" TECHNIQUE.
C
C      THE VARIABLE DEFINITIONS FOR THIS PROGRAM FOLLOW:
C      XK      NUMBER OF COMPONENTS IN SERIES
C      FAIL     NUMBER OF FAILURES PER COMPONENT
C      TIME     NUMBER OF MISSION UNITS EACH COMPONENT
C              IS UNDER TEST
C      BETA     OPTIMISTIC SCALE PARAMETER
C      ALFA     OPTIMISTIC SHAPE PARAMETER
C      XLAMS    SYSTEM FAILURE RATE
C      XLAMI    ESTIMATE OF THE COMPONENT FAILURE RATE
C      XLAMUP   UPPER CONFIDENCE LEVEL ON SYSTEM
C              FAILURE RATE
C
C      ALFA=0.5
6      XK=1.0
      XKPRI=1.469496
7      FAIL=0.0
8      BETA=50.0
9      TIME=30.0
10     C=0.0
      XLAMUP=0.0
      XLAMI=(ALFA+FAIL)/(TIME+BETA)
      XLAMS=XK*XLAMI
      C=(XK*(XLAMI/(TIME+BETA)))/XLAMS
      XLAMUP=((2.0*XLAMS)+((XKPRI**2)*C)+SQRT(((4.0*XLAMS)
1* (XKPRI**2)*C)+(XKPRI**4)*C**2))/2.0
101    WRITE(6,101)ALFA,FAIL,XK,TIME,BETA,XLAMUP
      FORMAT(' ',2X,'ALFA=',F4.1,2X,'FAIL=',F4.1,2X,'XK='
6F5.1,2X,'TIME=',F6.1,2X,'BETA=',F6.1,2X,'XLAMUP=',
7F10.6,/)
      TIME=TIME+20.0
102    IF(TIME.LE.51.0) GO TO 10
      TIME=TIME+30.0
103    IF(TIME.LE.101.0) GO TO 10
      BETA=BETA+50.0
104    IF(BETA.LE.101.0) GO TO 9
      FAIL=FAIL+1.0
      IF(FAIL.LE.1.1) GO TO 8
      XK=XK+1.0
      XKPRI=1.44015
      IF(XK.LE.2.1) GO TO 7
      XK=XK+2.0
      XKPRI=1.403713
      IF(XK.LE.5.1) GO TO 7
      XK=XK+2.0
      XKPRI=1.37725
      IF(XK.LE.10.1) GO TO 7
      XK=XK+15.0
      XKPRI=1.34092
      IF(XK.LE.30.5) GO TO 7
      ALFA=ALFA+0.5
      IF(ALFA.LE.1.1) GO TO 6
      STOP
      END

```



# COMPUTER PROGRAM FOR THE MODIFIED SEMI-CLASSIC METHOD

THIS PROGRAM IS A "MODIFIED SEMI-CLASSIC" IN THAT THE FAILURES ARE MODIFIED SO THAT ONLY ONE COMPONENT OR ONLY TWO COMPONENTS FAIL DURING THE COMPLETE SYSTEM TEST. ALL OTHER VARIABLES ARE THE SAME AS FOR THE "SEMI-CLASSIC" PROGRAM.

ONLY ONE SET OF VARIABLES ARE TESTED WITH THIS PROGRAM. THE VARIABLES TIME, BETA, ALFA, AND FAIL WOULD HAVE TO BE VARIED TO GENERATE A FULL TABLE OF VALUES.

C  
C  
C  
C  
C  
C  
C  
C  
C  
C

10

101

```
XKPRI=1.44015
XK=2.0
TIME=100.0
XLAMS=2.0/100.0
BETA=0.0
ALFA=0.0
C=1.0/TIME
XLAMUP=0.0
XLAMUP=(2.0*XLAMS+(XKPRI**2)*C+SQRT(((4.0*XLAMS)*(XKPRI**2)*C)+(XKPRI**4)*(C**2)))/2.0
WRITE(6,101)XK,BETA,TIME,XLAMUP
101  FORMAT(' ',2X,'XK = ',F7.3,2X,'BETA = ',F7.3,2X,
2  'TIME = ',F7.3,2X,'XLAMUP = ',F10.6,/)
XK=XK+3.0
XLAMS=2.0/100.0
IF(XK.LE.5.5) GO TO 10
XK=XK+2.0
XLAMS=2.0/100.0
IF(XK.LE.10.5) GO TO 10
XK=XK+15.0
XLAMS=2.0/100.0
IF(XK.LE.31.0) GO TO 10
STOP
END
```





# COMPUTER PROGRAM FOR THE BAYESIAN SIMULATION

ALFA = 1.0

THE DEFINITIONS OF THE VARIABLES USED IN THIS PROGRAM FOLLOW:

|       |                                                                        |
|-------|------------------------------------------------------------------------|
| TIME  | THE NUMBER OF MISSION UNITS THAT EACH COMPONENT IS UNDER TEST          |
| BETA  | THE OPTIMISTIC SCALE PARAMETER                                         |
| ALFA  | THE OPTIMISTIC SHAPE PARAMETER                                         |
| FAIL  | THE NUMBER OF TIMES THAT EACH COMPONENT WILL FAIL DURING THE TEST TIME |
| K     | THE NUMBER OF COMPONENTS THAT ARE IN THE SERIES SYSTEM                 |
| XLAM  | THE FAILURE RATE FOR AN INDIVIDUAL COMPONENT                           |
| XLAMS | THE SYSTEM FAILURE RATE                                                |

THIS SYSTEM IS CALLED  
DIMENSION XLAMS(1000),KEY(1000)

TIME=30.0

4 BETA=50.0

5 FAIL=0.0

7 IND = 1

600 DO 610 IO = 1,1000

XLAMS(IO) = 0.0

610 CONTINUE

ALFA=1.0

B=TIME+BETA

KA=ALFA+FAIL

GO TO (601,602,603,604,606,607),IND

601 K = 1

GO TO 608

602 K = 2

GO TO 608

603 K = 5

GO TO 608

604 K = 10

GO TO 608

606 K = 30

608 IX=999

DO 50 J=1,1000

DO 609 ILBDS = 1,K

TR=1.0

DO 20 I=1,KA

CALL RANDU(IX,IY,YFL)

IX=IY

TR=TR\*YFL

XLAM=-ALOG(TR)/B

XLAMS(J) = XLAMS(J) + XLAM

609 CONTINUE

KEY(J)=J

CONTINUE

CALL SHSORT(XLAMS,KEY,1000)

WRITE(6,60)K,TIME,BETA,FAIL,XLAMS(900)

FORMAT(' ',3X,'COMP. = ',I4,3X,'TIME = ',F7.2,3X,

1'BETA = ',F7.2,3X,'FAIL = ',F6.2,3X,'LAMS = ',

2F10.6,/) )

IND = IND + 1



```
GO TO 600
607 CONTINUE
   FAIL=FAIL+1.0
   IF(FAIL.LE.1.5) GO TO 7
   BETA=BETA+50.0
   IF(BETA.LE.100.1) GO TO 5
   TIME=TIME+20.0
   IF(TIME.LE.50.1) GO TO 4
   TIME=TIME+30.0
   IF(TIME.LE.100.1) GO TO 4
STOP
END
```



COMPUTER PROGRAM FOR THE  
MODIFIED BAYESIAN SIMULATION  
ALFA = 1.0

THIS BAYESIAN TECHNIQUE IS "MODIFIED" IN MUCH THE SAME WAY THAT THE SEMI-CLASSIC TECHNIQUE WAS IN THAT THE NUMBER OF FAILURES ARE "MODIFIED" SO THAT ONLY ONE AND ONLY TWO COMPONENTS WILL FAIL DURING THE SYSTEM TEST TIME

THE DEFINITIONS OF THE VARIABLES FOR THIS PROGRAM FOLLOW:

|       |                                                               |
|-------|---------------------------------------------------------------|
| TIME  | THE NUMBER OF MISSION UNITS THAT EACH COMPONENT IS UNDER TEST |
| BETA  | THE OPTIMISTIC SCALE PARAMETER                                |
| K     | THE NUMBER OF COMPONENTS THAT ARE IN THE SERIES SYSTEM        |
| JJ    | THE NUMBER OF COMPONENTS IN THE SYSTEM THAT FAIL              |
| XLAMS | THE SYSTEM FAILURE RATE                                       |

DIMENSION XLAMS(1000),KEY(1000)

1 DO 700 JJ=1,2

TIME=30.0

4 BETA=50.0

5 IND=1

KA IS EQUAL TO ALFA PLUS FAIL, WHICH IS 2  
THE INDEX "JJ" IS EQUAL TO THE NUMBER OF COMPONENTS THAT FAIL IN THE SERIES SYSTEM

600 DO 610 IO=1,1000

XLAMS(IO)=0.0

610 CONTINUE

B=BETA+TIME

GO TO (601,602,603,604,606,607),IND

601 K=1

GO TO 608

602 K=2

GO TO 608

603 K=5

GO TO 608

604 K=10

GO TO 608

606 K=30

608 IX=999

DO 80 J=1,1000

DO 24 JKJ=1,JJ

TR=1.0

10 DO 20 I=1,2

CALL RANDU(IX,IY,YFL)

IX=IY

20 TR=TR\*YFL

XLAMX=-ALOG(TR)/B

XLAMS(J)=XLAMS(J)+XLAMX

24 CONTINUE

IF(K-JJ.LE.0) GO TO 79

L=K-JJ

DO 30 ILBDS=1,L

TR=1.0

KA IS NOW EQUAL TO ONE FOR THE REST OF THE SYSTEM

DO 25 II=1,1

CALL RANDU(IX,IY,YFL)



```

IX=IY
25 TR=TR*YFL
   XLAM=-ALOG(TR)/B
   XLAMS(J)=XLAMS(J)+XLAM
30 CONTINUE
79 KEY(J)=J
80 CCNTINUE
   CALL SHSORT(XLAMS,KEY,1000)
   WRITE(6,90)K.TIME,BETA,XLAMS(900)
90  FORMAT(' ',3X,'CCMP. = ',14,3X,'TIME = ',F7.2,3X,
* 'BETA = ',F7.2,3X,'XLAMS = ',F10.6,/)
   IND=IND+1
   GO TO 600
607 CONTINUE
   BETA=BETA+50.0
   IF(BETA.LE.100.1) GO TO 5
   TIME=TIME+20.0
   IF(TIME.LE.50.1) GO TO 4
   TIME=TIME+30.0
   IF(TIME.LE.100.1) GO TO 4
700 CCNTINUE
   STOP
   END

```





COMPUTER PROGRAM FOR THE BAYESIAN SIMULATION  
ALFA = 0.5

THIS PROGRAM WILL GENERATE VARIATES FROM THE GAMMA DISTRIBUTION WHEN THE SHAPE PARAMETER (ALFA) IS LESS THAN 1.0

THE VARIABLE DEFINITIONS FOR THE MAIN PROGRAM FOLLOW:

|        |                                                                    |
|--------|--------------------------------------------------------------------|
| K      | THE OPTIMISTIC SHAPE PARAMETER                                     |
| XBETA  | THE OPTIMISTIC SCALE PARAMETER                                     |
| XTIME  | THE NUMBER OF MISSION UNITS THAT EACH COMPONENT IS UNDER TEST      |
| XFAIL  | THE NUMBER OF TIMES THAT EACH COMPONENT FAILS DURING THE TEST TIME |
| NCMP   | THE NUMBER OF COMPONENTS IN THE SERIES SYSTEM                      |
| XLAMS  | THE SYSTEM FAILURE RATE                                            |
| XLAMUP | THE UPPER CONFIDENCE LEVEL ON THE FAILURE RATE                     |

DIMENSION XLAMS(1000),KEY(1000)

REAL\*4 K

K=0.5

XTIME=30.0

30 XBETA=50.0

40 IND=1

600 DO 610 IO=1,1000

XLAMS(IO)=0.0

610 CONTINUE

BETA=1.0/(XBETA+XTIME)

GO TO(601,602,603,604,605,606),IND

601 NCMP=1

GO TO 607

602 NCMP=2

GO TO 607

603 NCMP=5

GO TO 607

604 NCMP=10

GO TO 607

605 NCMP=30

607 IX=999

CALL GMINIT(K,BETA)

10 DO 50 J=1,1000

DO 609 ILBDS=1,NCMP

CALL GAMA(IX,Z)

Z WILL = AN UNORDERED VALUE OF LAMBDAL

XLAMS(J)=XLAMS(J)+Z

609 CONTINUE

KEY(J)=J

50 CONTINUE

NOW ORDER THE SYSTEM FAILURE RATES

CALL SHSORT(XLAMS,KEY,1000)

WRITE(6,60)NCMP,XBETA,XTIME,XLAMS(900)

60 FORMAT(' ',2X,'NCMP =',I4,2X,'BETA =',F6.2,2X,

\* 'TIME =',F6.2,2X,'XLAMS =',F10.6,/)

IND=IND+1

GO TO 600

606 CONTINUE

XBETA=XBETA+50.0



```
IF(XBETA.LE.100.1) GO TO 40
XTIME=XTIME+20.0
IF(XTIME.LE.50.1) GO TO 30
XTIME=XTIME+30.0
IF(XTIME.LE.100.1) GO TO 30
END
```



SUBROUTINE GMINIT(K,BETA)

ADDITIONAL ENTRY POINTS:

RESULT  
GAMA(IX,Z)

PURPOSE:

GENERATION OF GAMMA RANDOM DEVIATES WITH SHAPE  
PARAMETER LESS THAN ONE.

METHOD:

A MODIFICATION OF MARSAGLIA'S BOX-WEDGE-TAIL METHOD  
FOR NORMAL DEVIATES IS USED. THE PDF IS DECOMPOSED  
INTO A HEAD REGION, A NUMBER (DEPENDENT ON K) OF  
RECTANGLES AND WEDGES AND A TAIL REGION. THE GMINIT  
SECTION OF THE SUBROUTINE ALSO SETS UP A BINARY  
SEARCH TREE TO BE USED FOR EFFICIENT SELECTION OF THE  
PROPER REGION DURING THE ACTUAL GENERATING PROCESS,  
WHICH IS HANDLED BY THE GAMA SECTION

DESCRIPTION OF PARAMETERS

K        GAMMA DISTRIBUTION SHAPE PARAMETER  
          (MUST BE .GE. 0.05 AND .LE. 1.0)  
BETA     GAMMA DISTRIBUTION SCALE PARAMETER  
IX       SEED FOR RANDOM NUMBER GENERATOR  
Z        RETURNED GAMMA DEVIATE

THE PDF OF THE GAMMA FUNCTION IS GIVEN BY

$$F(X) = (1/BETA)^{*}K * X^{*(K-1)} * EXP(-X/BETA)/GAMMA(K)$$

THE FOLLOWING SUBROUTINES ARE USED:

RANDOM(IX)    RETURNS A UNIFORM (0,1) DEVIATE  
INVGAM(K,X)   COMPUTES THE INVERSE GAMMA CDF  
IGAM(K,X)     COMPUTES THE INCOMPLETE GAMMA  
              FUNCTION(GAMMA CDF)

NOTE:

UNDERFLOW IS POSSIBLE WHEN K IS LESS THAN .18 AND  
BECOMES MORE LIKELY AS K DECREASES. WHEN K IS 0.5  
THE PROBABILITY OF UNDERFLOW IS ABOUT .000129 FOR  
ANY GENERATED DEVIATE.

SUBROUTINE GMINIT(K,BETA)

REAL\*4 K,INVGAM,IGAM

INTEGER\*4 FIRST,TABLE,BOTTOM,END

LOGICAL\*1 USED

DIMENSION P(100),X(101),H(100),Q(100),R(100),B(100)

DIMENSION TABLE(202),PROB(202),NEXT(202),LAST(202)

DIMENSION TEST(202),LIST(202),USED(202)

DIMENSION RAND(2)

EQUIVALENCE (U,RAND(1)),(V,RAND(2))

THIS FIRST SECTION INITIALIZES CONSTANTS AND TABLES TO  
BE USED BY GAMA WHEN IT IS CALLED. THE FOLLOWING  
ARE USED BY GAMA:

P0        PROBABILITY FOR "HEAD" REGION  
PN        PROBABILITY FOR "TAIL" REGION  
P(I)      PROBABILITY FOR I-TH RECTANGLE  
X(I)      LEFT-HAND BOUNDARY OF I-TH RECTANGLE  
H(I)      WIDTH OF I-TH RECTANGLE  
Q(I)      PROBABILITY OF I-TH WEDGE





```

C      R(I)    REJECTION TEST RATIO FOR I-TH WEDGE
C      B(I)    Y INTERCEPT FOR I-TH WEDGE
C      ALPHA   SHAPE PARAMETER - 1.
C      PROB    ORDERED VECTOR OF PROBABILITIES
C      TABLE  VECTOR OF WEDGE/RECTANGLE NUMBERS CORRESPOND-
C              ING TO PROBABILITIES IN PROB
C      FIRST   STARTING POINT FOR BINARY SEARCH
C      NEXT,   LINKS FOR BINARY SEARCH
C      LAST
C      J1      POSITION IN PROB OF P0
C      H1 TO   CONSTANTS FOR APPROXIMATION TO INVERSE GAMMA
C      H4      CDF FOR SMALL VALUES OF Z
C
C      CHECK FOR K IN RANGE
C      IF((K.GE.0.05) .AND. (K.LE.1.0))GO TO 5
C      WRITE(6,4)K
4     FORMAT(//'OGMINIT CALLED WITH K=',1PE16.6,
C      *' OUT OF RANGE'/)
C      RETURN
C
C      GET UPPER BOUND ON NUMBER OF RECTANGLES
5     N=20.+6.6/K
C      IF(N.GT.100)N=100
C      M=2*N+2
C      MM=M-1
C      ALPHA=K-1.
C      GK=GAMMA(K)
C      P0=5.E-5/(K*K)
C      HFAC=2.
C
C      SET UP RECTANGLE BOUNDS
C      X(1)=INVGAM(K,P0)
C      P0=IGAM(K,X(1))
C      H(1)=.25*X(1)
C      DO 10 I=2,N
C          X(I)=X(I-1)+H(I-1)
C          H(I)=H(I-1)*HFAC
C          P(I)=0.
C          Q(I)=0.
10    CONTINUE
C      X(N+1)=X(N)+H(N)
C
C      ZERO PROBABILITY VECTORS AND LINKS
C      DO 15 I=1,M
C          NEXT(I)=0
C          LAST(I)=0
C          PROB(I)=0.
C          LIST(I)=0
C          USED(I)=.FALSE.
15    CONTINUE
C
C      FIND COEFFICIENTS FOR NEWTON-RAPHSON APPROXIMATION TO
C      SLOPE OF PDF
C      B1=-2.*ALPHA
C      B2=ALPHA*(ALPHA-1.)
C      A1=B1+1.
C      A2=ALPHA*(ALPHA-2.)
C      C=1.-ALPHA
C
C      FIND RECTANGLE PROBABILITIES AND WEDGE VALUES
C      PL=P0
C      FL=EXP(ALPHA*ALOG(X(1))-X(1))/GK
C      DO 40 I=1,N
C          FU=EXP(ALPHA*ALOG(X(I+1))-X(I+1))/GK
C          PU=IGAM(K,X(I+1))
C          P(I)=H(I)*FU
C          Q(I)=PU-PL-P(I)
C

```



```

C      NEWTON-RAPHSON ITERATION TO FIND POINT WHERE
C      SLOPE OF PDF IS PARALLEL TO CHORD
C
      W=X(I)
      S=(FU-FL)/H(I)
      SC=S*GK
      DO 20 J=1,8
          Y=W*((W+A1)*W+A2+SC*EXP(C*ALOG(W)+W))/((W+B1
              IF(ABS(Y-W).LT.1.E-4*Y)GO TO 30
          W=Y
20      CONTINUE
C
C      FIND VALUES FOR REJECTION METHOD
C
30      B(I)=EXP(ALPHA*ALOG(Y)-Y)/GK+S*(X(I)-Y)
      R(I)=(B(I)-FU)/(FL-FU)
C
C      TEST TO SEE IF ENOUGH RECTANGLES HAVE BEEN TAKEN
C      IF(PU.GT.0.999)GO TO 45
C
C      RESET PROBABILITY AND FUNCTION VALUES FOR NEXT
C      RECTANGLE
      PL=PU
      FL=FU
40      CONTINUE
C
C      FIND LOWER END OF NCN-ZERO PROBABILITY VECTOR
45      LOW=2*(N-1)+1
C
C      FIND TAIL PROBABILITY
      SUM1=P0
      DO 50 J=1,N
          SUM1=SUM1+P(J)+Q(J)
50      CONTINUE
      PN=1.-SUM1
C
C      GET VALUES TO COMPUTE LOW END APPROXIMATION TO INVERSE
C      GAMMA CDF
      H1=K*GK*P0
      H2=1./K
      H3=(K+1.)*5.0E-7
      H4=4./(K+1.)
C
C      GENERATE PROBABILITY VECTOR
C
      DO 80 I=1,N
          PROB(I)=P(I)
          TABLE(I)=I
          PROB(I+N)=Q(I)
          TABLE(I+N)=-I
80      CONTINUE
      PROB(M-1)=P0
      TABLE(M-1)=0
      PROB(M)=PN
      TABLE(M)=0
C
C      SORT PROBABILITY VECTOR
C
      DO 110 I=1,MM
          ICH=0
          L=M-I
          DO 100 J=1,L
              IF(PROB(J)-PROB(J+1))100,100,90
90              TEMP=PROB(J)
              PROB(J)=PROB(J+1)
              PROB(J+1)=TEMP
              ITEMP=TABLE(J)
              TABLE(J)=TABLE(J+1)
              TABLE(J+1)=ITEMP
          ICH=1
          100
          110

```



```

100      CONTINUE
        IF(ICH)120,120,110
110  CCNTINUE
    PROB(M)=1.
C
C  CCNVERT PROB TO CUMULATIVE PROBABILITIES AND
C  FIND FIRST AND J1
120  J1=0
    FIRST=0
    L=LOW+1
    DO 130 I=L,M
        IF((TABLE(I).EQ.0) .AND. (PROB(I).EQ.P0))J1=I
        PROB(I)=PROB(I)+PROB(I-1)
        IF((PROB(I).GE.0.5) .AND. (FIRST.EQ.0))FIRST=I
130  CONTINUE
    IF(FIRST.EQ.M)FIRST=MM
C
    IF(J1)140,140,150
140  J1=LOW
C
C  NOW DETERMINE THE VECTORS NEXT AND LAST FOR BINARY
C  SEARCH
150  BOTTOM=1
    END=1
    PR=.25
    LIST(1)=FIRST
    TEST(1)=.5
    USED(FIRST)=.TRUE.
151  DO 159 I=1,BOTTOM
        LI=LIST(I)
        IF(USED(LI+1))GO TO 155
        PRN=TEST(I)+PR
        DO 152 J=LOW,MM
            IF(PROB(J).GT.PRN)GO TO 153
152      CONTINUE
153      IF(.NOT. USED(J))GO TO 154
        J=J-1
        IF(LI-J)153,155,155
154      NEXT(LI)=J
155      LIST(I)=NEXT(LI)
        IF((LI.EQ.LOW) .OR. USED(LI-1))GO TO 159
        PRL=TEST(I)-PR
        TEST(I)=PRN
        DO 156 J=LOW,MM
            IF(PROB(J).GT.PRL)GO TO 157
156      CONTINUE
157      IF(.NOT. USED(J))GO TO 1571
        J=J+1
        IF(LI-J)158,158,157
1571     LAST(LI)=J
        GO TO 1585
158     J=LI
1585     END=END+1
        LIST(END)=J
        TEST(END)=PRL
159  CONTINUE
    BOTTCM=END
    PR=PR*0.5
    I=1
1591 IF(LIST(I))160,160,163
160  BOTTCM=BOTTOM-1
    IF(BOTTOM)165,165,161
161  DO 162 J=I,BOTTCM
        LIST(J)=LIST(J+1)
        TEST(J)=TEST(J+1)
162  CONTINUE
    GO TO 1591
163  USED(LIST(I))=.TRUE.
    I=I+1
    IF(I.LE.BOTTOM)GO TO 1591

```





```

END=BOTTCM
GO TO 151

C
C
C 165 SETUP FIRST CALL TO RANDOM
CALL OVFLOW

C
C
C THROUGH WITH SETUP PHASE. QUIT.
RETURN

C
C
C THIS SECTION PRINTS OUT THE VALUES GENERATED BY GMINIT

ENTRY RESULT
WRITE(6,166)K,BETA
166 FORMAT(/'1GENERATED VALUES FOR K=',1PE14.6,
* ' BETA=',E14.6)
WRITE(6,170)P0,PN
170 FORMAT('0HEAD PROBABILITY=',1PE15.6,
* ' TAIL PROBABILITY=',E15.6)
WRITE(6,180)
180 FORMAT('0RECTANGLE/WEDGE VALUES'//2X,'I',9X,'X(I)',12X
* 'P(I)',12X,'Q(I)',12X,'R(I)',12X,'B(I)'//)
SUM1=0.
SUM2=0.
DO 192 I=1,N
IF(P(I))193,193,185
185 SUM1=SUM1+P(I)
SUM2=SUM2+Q(I)
WRITE(6,190)I,X(I),P(I),Q(I),R(I),B(I)
190 FORMAT(1X13,1P5E16.6)
192 CONTINUE
193 WRITE(6,194)SUM1,SUM2
194 FORMAT('0SUMS',15X,1P2E16.6)
WRITE(6,195)J1,H1,H2,H3,H4
195 FORMAT(/'0VALUES FOR HEAD/TAIL APPROXIMATION:'/' J1=',
* I6/' H1=',E16.6,' H2=',E16.6,' H3=',E16.6,' H4',
1E16.6)
WRITE(6,196)FIRST
196 FORMAT(/'0STARTING POINT FOR BINARY SEARCH',I4)
WRITE(6,197)(I,PROB(I),TABLE(I),NEXT(I),LAST(I),I=1,M)
197 FORMAT(/'0PROBABILITY VECTOR AND LINKS'//14X,'PROB',9X
* 'TABLE',4X,'NEXT',5X,'LAST'//(1X15,1PE16.6,3P3I9))
RETURN

C
C
C THIS IS THE SECTION THAT ACTUALLY COMPUTES THE RANDOM
VARIATES

ENTRY GAMA(IX,Z)

C
C
C GET TWO UNIFORM DEVIATES
CALL RANDOM(IX,U,2)

C
C
C CONDUCT BINARY SEARCH OF PROBABILITY VECTOR

J=FIRST
200 IF(U-PROB(J))210,250,230
210 IF(LAST(J))250,250,220
220 J=LAST(J)
GO TO 200
230 IF(NEXT(J))245,245,240
240 J=NEXT(J)
GO TO 200
245 J=J+1

C
C
C LOCATED PROBABILITY DIVISION. GET TABLE VALUE.
250 N=TABLE(J)
IF(N)260,290,320

C
C
C THIS SECTION IS FOR THE WEDGES

260 N=-N
CALL RANDOM(IX,U,1)

```



```

270 IF(U.LE.V)GO TO 280
    TEMP=U
    U=V
    V=TEMP
280 Z=X(N)+H(N)*U
    IF(V.LE.R(N))GO TO 330
C
C    THIS STEP IS PERFORMED VERY RARELY
    W=U+EXP(ALPHA*ALOG(Z)-Z)/B(N)
    IF(V.LE.W)GO TO 330
    CALL RANDOM(IX,U,2)
    GO TO 270
C
C    THIS SECTION IS FOR HEAD/TAIL PROBABILITIES
290 IF(J.EQ.J1)GO TO 300
    Z=INVGAM(K,1.-PN*V)
    GO TO 330
300 Z=(H1*V)**H2
    IF(Z.LT.H3)GO TO 330
    Z=2.*Z/(1.+SQRT(1.-H4*Z))
    GO TO 330
C
C    THIS SECTION IS FOR THE RECTANGLES
320 Z=X(N)+H(N)*V
C
C    SCALE GAMMA VARIATE AND EXIT
330 Z=Z*BETA
    RETURN
    END

```



C      INVGAM SUBROUTINE

```
FUNCTION INVGAM(K,Z)
REAL*4 INVGAM,IGAM,K,EPS/1.E-08/
IF(Z.GT.0.0)GO TO 10
INVGAM=0.0
RETURN
10 X=Z
T=K-1.
G=GAMMA(K)
DO 40 I=1,30
Y=G*(IGAM(K,X)-Z)/(EXP(T*ALOG(X)-X))
20 IF(Y.LT.X)GO TO 30
Y=Y*.5
GO TO 20
30 X=X-Y
IF(ABS(Y)-EPS*X)50,50,40
40 CONTINUE
50 INVGAM=X
RETURN
C      THIS IS THE END OF THE INVGAM SUBROUTINE
END
```



C IGAM SUBROUTINE

```
FUNCTION IGAM(K,X)
IMPLICIT REAL*8 (D)
REAL*4 IGAM,K
REAL*8 EPS/1.D-13/
IF(X.GT.0.)GO TO 5
IGAM=0.
RETURN
5 IF(X.LE.12.0)GO TO 8
IGAM=1.0
RETURN
8 DX=DBLE(X)
CK=DBLE(K)
DTERM=DX**CK/DGAMMA(CK)
DSUM=DTERM/CK
DO 20 I=1,30
IF(DABS(DTERM)-EPS*DSUM)40,40,10
10 DTERM=DTERM*(-DX)/DFLOAT(I)
DSUM=DSUM+DTERM/(CK+DFLOAT(I))
20 CONTINUE
40 IGAM=SNGL(DSUM)
RETURN
C THIS IS THE END OF THE IGAM SUBROUTINE
END
```





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ABSTRACT

A comparison of several techniques is presented for determining upper confidence levels for a system failure rate. A series system of components with exponential failure rates is examined. Classical computational techniques are compared with Bayesian techniques in determining the upper confidence level of a system failure rate. A sensitivity analysis is conducted on several of the parameters as part of the comparison.





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